AIAA 81-4029

Simple, Numerically Generated Orthogonal Coordinate System for Viscous Flow

R. A. Graves Jr.* and H. H. Hamilton II†
NASA Langley Research Center, Hampton, Va.

Abstract

A SIMPLE, numerically generated orthogonal coordinate system was developed[1] and applied[2] to inviscid blunt-body flow for a wide range of conditions. This orthogonal coordinate generator has been extended to viscous flows through the use of a highly stretched mesh and fourth-order differencing for the numerical determination of the metric coefficients. In the physical plane, the highly stretched mesh allows for resolution of the boundary layer near the body's surface, yet in the rectangular computational plane, the mesh is equally spaced. Since this equally spaced orthogonal mesh intersects the shock wave and body surface normally, both the finite differences and boundary conditions are simplified. The full Navier-Stokes equations are solved using Stetter's three-step numerical technique[3,4] for laminar viscous compressible supersonic flow over axisymmetric blunt bodies with varying degrees of bluntness, including reverse curvature. Results indicate that the coordinate system performed well and no problems were encountered in the coupling of the numerical coordinate generator with the fluid dynamic equations.

Contents

Much effort has been expended to develop coordinate transformations and/or mesh generators for varying degrees of geometric complexity. For viscous flow over blunted bodies, the large gradients adjacent to the surface must be represented accurately by the finite-difference approximations to the governing equations, and toward this goal almost all numerical solutions to date for blunt body flows have used relatively simple geometries conducive to use with natural or nearly orthogonal coordinate systems. The present analysis uses a coordinate generation technique[1] along with time-dependent solution concepts[2,3] to obtain solutions to the Navier-Stokes equations for viscous compressible flow over blunt bodies. The emphasis is on obtaining fluid-flow solutions for blunt bodies of varying degrees of geometric complexity where the mesh must be highly compressed to resolve the thin viscous boundary layer.

The numerically generated orthogonal coordinates will be determined from the original Cartesian coordinate system's description of the body surface and bow shock. The surface distance along the body forms one of the transformed orthogonal coordinates and is defined as zero on the axis of symmetry and increases to a value of unity at the end point of the forebody surface. The normal distance starts at zero on the body's surface and increases to unity on the shock wave so that in the rectangular, transformed computational plane, the normalized distances in both coordinate directions vary between zero and one. The level lines between the outer boundary and the body are constructed along straight lines connecting corresponding points on the body and shock wave. The spacing of the level lines is arbitrary; however, for viscous flows, the boundary layer must be resolved, necessitating the use of a mesh stretcher.[5] Once the level lines have been determined, the normal lines are constructed numerically[6] so that an orthogonal system is defined.[1] Starting on the body surface, the solution proceeds point by point along a level line until all normals on that level have been determined. Then the solutions proceeds to the next level and the process is continued until the outer boundary is reached. Once the coordinate system is constructed, the metric coefficients can be determined numerically. Since the computational plane is equally spaced, the derivatives for the metric coefficients can be evaluated using equally spaced central finite differences. For the present analysis, fourth-order-accurate relations[7] are used in place of the simpler second-order-accurate finite differences in order to produce smoothly varying metric coefficients adjacent to the axis of symmetry. These fourth-order-accurate metric coefficients proved to be totally satisfactory for the viscous analysis even in regions of high mesh stretching.

The full Navier-Stokes equations are solved on the coordinate mesh using Stetter's three-step numerical technique.[3,4] A fourth-order numerical damping[8] of the solution is performed to prevent oscillations in the flow from destroying the solution process.[9] The coordinate system was re-evaluated only at the end of the third step in the numerical solution procedure. This process proved entirely satisfactory even though no real optimization of the fluid dynamics/coordinate system calculation procedure was attempted. The present analysis was used to calculate the flow over blunt bodies with increasing nose bluntness. A family of bodies having a range of bluntness, including reverse curvature, was generated using

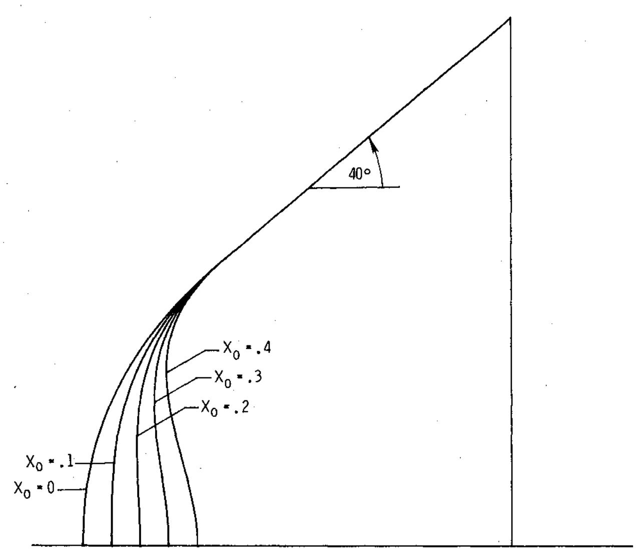

Fig. 1 Forebody shapes as a function of X_0.

Received Feb. 26, 1980; synoptic received July 14, 1980. This paper is declared a work of the U. S. Government and therefore is in the public domain. Full paper available from National Technical Information Service, Springfield, Va. 22151, as N80-00000 at the standard price (available upon request). Accession number not available, order by title and author.

*Research Leader, Aerothermodynamics Branch, Space Systems Division. Member AIAA.

†Aero-Space Technologist, Aerothermodynamics Branch, Space Systems Division.

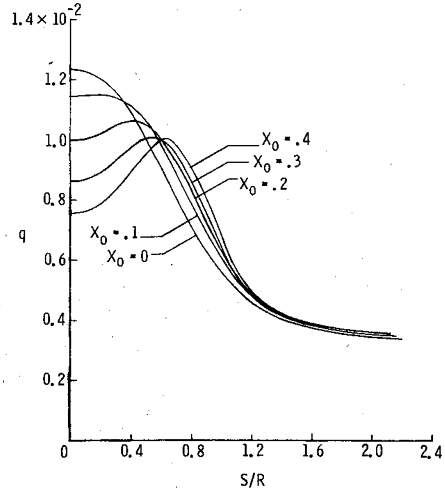

Fig. 2 Effect of nose blunting on the heat-transfer distribution.

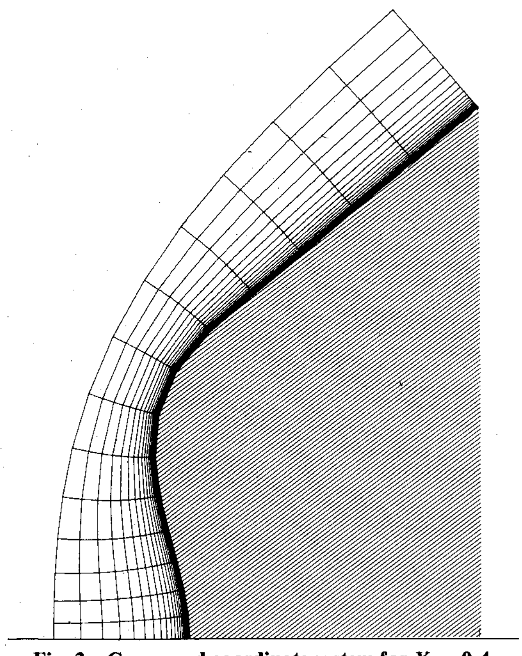

Fig. 3 Converged coordinate system for $X_0 = 0.4$.

the following cubic forebody generator:

$$X = X_0 + a_1 y^2 + a_2 y^3$$

The coefficient X_0 determines the nose offset while the coefficients a_1 and a_2 are determined such that the forebody nose section joins smoothly to the conical flank section with the specified conical surface angle. Figure 1 shows the family of body shapes run using the present analysis and Fig. 2 gives the surface heat-transfer rates for the following conditions: $M_\infty = 10.33$, $\gamma = 1.4$, $P_\infty = 100.77$ N/m^2, $T_\infty = 46.26$ K, and $Np_r = 0.7$. A typical converged coordinate system is given in Fig. 3. All of the results obtained in the present analysis demonstrate the simplicity and ease of application of the coordinate generator for viscous flow with no noted undesirable flowfield/coordinate system coupling effects. The success of the present analysis is due in large part to the smooth metric coefficients produced by the fourth-order differencing.

References

[1]Graves, R. A., Jr., "Application of a Numerical Orthogonal Coordinate Generator to Axisymmetric Blunt Bodies," NASA TM 80131, Oct. 1979.

[2]Hamilton, H. H., II and Graves, R. A., Jr., "Application of a Numerically Generated Orthogonal Coordinate System to the Solution of Inviscid Axisymmetric Supersonic Flow Over Blunt Bodies," NASA TP 1619, Jan. 1980.

[3]Graves, R. A., Jr. and Johnson, N. E., "Navier-Stokes Solutions Using Stetter's Method," *AIAA Journal,* Vol. 16, Sept. 1978, pp. 1013-1015.

[4]Kumar, A. and Graves, R. A., Jr., "Comparative Study of the Convergence Rates of Two Numerical Techniques," *AIAA Journal,* Vol. 16, Nov. 1978, pp. 1214-1216.

[5]Blottner, F. G., "Variable Grid Scheme Applied to Turbulent Boundary Layers," *Computational Methods in Applied Mechanics and Engineering,* Vol. 4, Sept. 1974, pp. 179-194.

[6]McNally, W. D., "FORTRAN Program for Generating a Two-Dimensional Orthogonal Mesh Between Two Arbitrary Boundaries," NASA TN D-6766, 1972.

[7]Milne, W. E., *Numerical Calculus,* Princeton University Press, Princeton, N. J., 1949.

[8]Barnwell, R. V., "A Time-Dependent Method for Calculating Supersonic Angle of Attack Flow About Axisymmetric Blunt Bodies with Sharp Shoulders and Smooth Nonaxisymmetric Blunt Bodies," NASA TN D-6283, Aug. 1971.

[9]Kumar, A. and Graves, R. A., Jr., "Numerical Solution of the Viscous Hypersonic Flow Past Blunted Cones at Angle of Attack," AIAA Paper 77-172, Jan. 1977.